NASA CR-159955 October 20, 1978

(NASA-CR-159955) GLOBAL CRUSTAL RESPONSE MODEL Semiannual Report, 3 Apr. - 3 Oct. 1978 (Business and Technological Systems, Inc.) 22 p HC A02/MF A01 CSCL 08G

N80-15705

Unclas

G3/46 45811

SEMI-ANNUAL REPORT FOR GLOBAL CRUSTAL RESPONSE MODEL

Contract NAS 5-25010

For the Period April 3, 1978 through October 3, 1978

Submitted to

NASA

Goddard Space Flight Center

Greenbelt, Maryland 20771

By

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TABLE OF CONTENTS

		Page
1.0	SUMMARY OF WORK ACCOMPLISHED	1
2.0	WORK PERFORMED DURING REPORTING PERIOD	2
2.1	Body Tide Program	2
2.2	Earth Response to Ocean Loading	9
2.3	Conformance to Schedule	14
2.4	Analysis of Progress	14
2.5	Efforts to Achieve Reliability	14
2.6	Personnel Changes	14
2.7	New Technology	16
3.0	WORK PLANNED FOR NEXT REPORTING PERIOD	17
4.0	FISCAL AND PLANNING DATA	18

1.0 SUMMARY OF WORK ACCOMPLISHED

A computer program has been developed for calculating the radial displacement due to the body tide as a function of spatial position and time. The positions of the moon and sun are evaluated by means of the Hill-Brown and Newcomb theories, respectively.

Utilizing Green's functions tabulated by Farrell for the elastic earth responses due to point loads, a convolution program has been developed to evaluate the elastic earth responses due to ocean loading by a given ocean tide constituent. These responses include radial and horizontal displacement, tilt and gravity acceleration, and strain tensor components.

Using the convolution programs, the global radial displacement due to the M_2 tide has been obtained.

2.0 WORK COMPLETED DURING REPORTING PERIOD

The tide of the solid earth is composed of two basic responses; (1) a body tide due to the yielding of the earth to direct forces of the sun and moon and (2) a "load" tide produced by surface forces from the time varying ocean tides. It is difficult to distinguish between these two responses because their time dependence is similar, being driven by the same ultimate force. However, the nature of the driving force of the "body tide" is well understood, while the knowledge of the deep ocean tides through global numerical modeling is a recent advancement.

The body tide varies in a relatively smooth nature over the earth's surface, depending principally on averaged overall elastic properties while the load tide is complicated by discontinuities of the surface load at coastal boundaries and by local ocean tide circulation (e.g., amphidromes and anti-amphidromes). Moreover, the displacement of the load tide is appreciable only in the crust and upper mantle, while the body tide has relatively large amplitude through most of the earth's interior. The load tide response then depends more on local crustal properties so that variations in near surface earth structure will be more reflected in the load tide.

The work performed during the first six months consisted of implementing an analytic program for representing the body tide deformation, and establishing the required software for evaluating the responses of the earth to the ocean loading tide. These tasks will be discussed separately in the following sections.

2.1 Body Tide Program

The body tide elevation is given by

$$U_B\ (\phi,\ \lambda;\ t) = \frac{M_d R_e}{M_e} \sum_{n=2}^{\infty} h_n \left(\frac{R_e}{R_d}\right)^{n+1} P_n(\cos\Theta_{MS})$$

where M_d is the mass of the disturbing body (Moon or Sun), R_d is the geocentric distance to the body and h_n are Love numbers. The angle Θ_{MS} denotes the geocentric zenith angle of the moon (sun) from the point of elevation. The terms in the expansion fall off rapidly so only the first term is of major significance.

A computer program has been developed to evaluate the solid earth tide (body tide) at a user specified position on the earth for a desired time interval; the algorithm presently evaluates for a one day period at one hour increments. Required input are the Modified Julian Day, and the geodetic coordinates of the point of interest.

The radial body tide, U_B , is evaluated as

$$U_B \quad (\phi,\lambda;t) = \frac{M_d}{M_e}\left(\frac{R_e}{R_d}\right)^3 \frac{R_e h_2}{2} \left[3(\hat{R}_d \cdot \hat{r})^2 - 1\right]$$

where $\hat{r}$ represents the unit radius vector at the point of interest on the earth:

$$\hat{r} = [\cos\phi\cos\lambda,\ \cos\phi\sin\lambda,\ \sin\phi]$$

where geodetic latitude, ϕ , and longitude, λ , are program inputs.

$\hat{R}_d$ represents the unit vector from the center of the earth in the direction of the disturbing body:

$$\hat{R}_d = [\lambda', \mu', \nu']$$

where λ', μ', ν' give the position in earth-fixed coordinates.

In calculating $\hat{R}$ for the moon, a true longitude and the latitude (above the plane of the ecliptic) are derived from the Hill-Brown theory using the Modified Julian Day. Brown's tables express the coordinates of the moon as sums of periodic terms whose arguments are algebraic sums of the multiples of ℓ, ℓ', F, D, Γ . See Tables I and II.

TABLE I

Ecliptic Elements

MJD = Modified Julian Day

D = MJD-2415020.0

D1 = D*1.E-4

ℓ = 296.104608+13.0649924465*D+0.0006889*$(D1)^2$

ℓ' = 358.475845+0.9856002670*D-0.0000112*$(D1)^2$

F = 11.250889+13.2293504490*D-0.0002407*$(D1)^2$

D = 350.737486+12.1907491914*D-0.0001076*$(D1)^2$

Γ = 281.220833+0.470684*D1+0.339E-4*$(D1)^2$

ε = 23.452294-0.0035626*D1-0.123E-6*$(D1)^2$

θ_g = 99.6904833+360.98564733*D-180.0

Table II Development of Lunar Position

Coeff of sine in $\delta\lambda_m$ (Seconds of arc)	Multiples of ℓ	ℓ'	F	D	Γ
22639.5	1	0	0	0	0
-4586.426	1	0	0	-2	0
2369.902	0	0	0	2	0
769.016	2	0	0	0	0
-668.111	0	1	0	0	0
-411.608	0	0	2	0	0
-211.656	2	0	0	-2	0
-205.962	1	1	0	-2	0
-125.154	0	0	0	1	0
191.953	1	0	0	2	0
-165.145	0	1	0	-2	0
147.693	1	1	0	0	0
-109.667	1	1	0	0	0
Coeff in sine in Latitude, ϕ_m (Seconds of arc)					
18461.48	0	0	1	0	0
1010.180	1	0	1	0	0
-999.695	-1	0	1	0	0
-623.658	0	0	1	-2	0
117.262	0	0	1	2	0
199.485	-1	0	1	2	0
-166.577	1	0	1	-2	0
61.913	2	0	1	0	0

Table III Development of *sin* λ_s and *cos* λ_s

Coef. × 10^5 of cosine in *cos* λ_s and of sine in *sin* λ_s	Multiples of				
	ℓ	ℓ'	F	D	Γ
99972	0	1	0	0	1
1674	0	2	0	0	1
32	0	3	0	0	1
1	0	4	0	0	1
2	0	1	0	1	1
-1675	0	0	0	0	1
-4	0	-1	0	0	1
-2	0	1	0	-1	1
4	0	0	1	-1	0
-4	0	2	-1	1	2

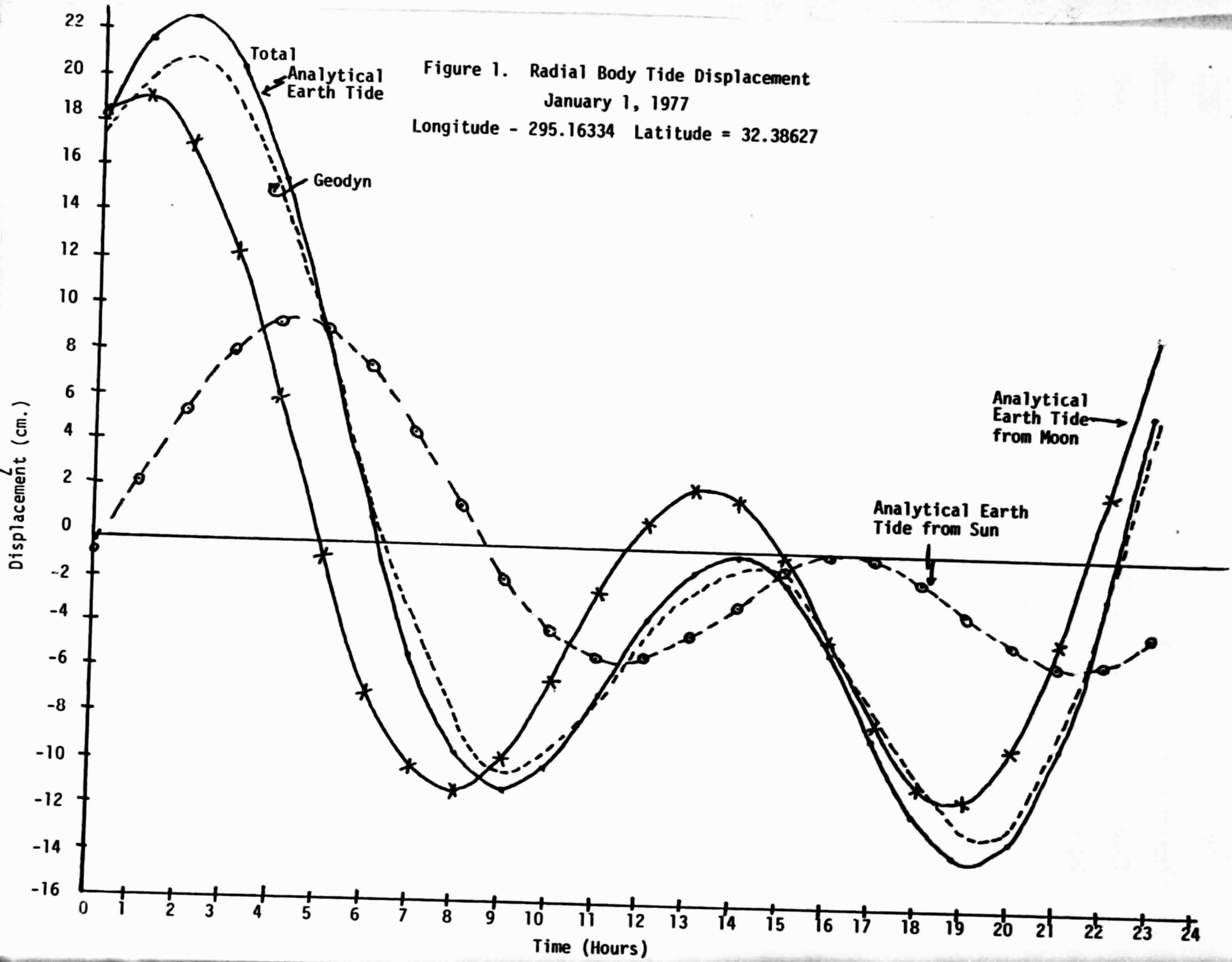


Figure 1. Radial Body Tide Displacement
January 1, 1977
Longitude - 295.16334 Latitude = 32.38627

The derived ecliptic lunar position (λ_m, ϕ_m) is converted to inertial coordinates

$$\lambda' = \cos\lambda_m \cos\phi_m$$

$$\mu' = \sin\lambda_m \cos\phi_m \cos\varepsilon - \sin\phi_m \sin\varepsilon$$

$$\nu' = \sin\phi_m \cos\varepsilon + \sin\lambda_m \cos\phi_m \sin\varepsilon$$

where ε is the obliquity to the ecliptic and

$$\lambda_m = \ell' + D + \Gamma + \delta\lambda_m \quad .$$

The conversion to earth-fixed coordinates is accomplished by a matrix transformation

$$\left(\hat{R}_d\right)_{EF} = \left(M\hat{R}_d\right)_{\text{Inertial}} \qquad (1)$$

where

$$M = \begin{pmatrix} \cos\Theta_g & \sin\Theta_g & 0 \\ -\sin\Theta_g & \cos\Theta_g & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and Θ_g is the Greenwich hour angle (Table I).

The unit vector $\hat{R}_d$ in the direction of the sun is derived from Newcomb's theory in the same manner. The ecliptic elements, ℓ, ℓ', F, D, Γ, are the same as for the lunar development. The solar coordinates $(\cos\lambda_s, \sin\lambda_s)$ are expressed as algebraic sums as listed in Table III.

The conversion to equatorial coordinates is

$$\lambda' = \cos\lambda_s$$

$$\mu' = \sin\lambda_s \cos\varepsilon$$

$$\nu' = \sin\lambda_s \sin\varepsilon$$

and a final transformation to earth-fixed coordinates is applied, using Equation (1).

A comparison of the radial body tide displacement computed by this analytic procedure and the ephemeris calculation from GEODYN is displayed in Figure 1.

2.2 Earth Response to Ocean Loading

The calculation of the earth deformation due to surface mass loads closely follows the Green's function approach of Longman and Farrell. Farrell has integrated the equations of motion for a self-gravitating elastic spherical earth using a Gutenberg-Bullen A earth model and produced load Love numbers h_n', l_n', and k_n' to high order n. The elastic earth response then reduces to a convolution of the ocean tide with the Green's function over the global ocean. The appropriate Green's functions for the augmented potential and surface vertical and horizontal displacements at an angular distance γ (spherical earth) from a point load at the pole per unit of loading mass are

$$\Phi'(\gamma) = \frac{Rg}{M_e} \sum_{n=0}^{\infty} k_n' P_n(\cos\gamma)$$

$$U'(\gamma) = \frac{R}{M_e} \sum_{n=0}^{\infty} h_n' P_n(\cos\gamma)$$

$$V'(\gamma) = \frac{R}{M_e} \sum_{n=1}^{\infty} l_n' \frac{\partial P_n(\cos\lambda)}{\partial\gamma}$$

where M_e is the mass of the earth, R is the mean earth radius and g is the acceleration of gravity at the surface. Green's functions for the deflection of the local vertical and the differential gravity acceleration are

$$T'(\gamma) = -\frac{1}{M_e} \sum_{n=0}^{\infty} (1+k_n'-h_n') \frac{\partial P_n(\cos\gamma)}{\partial\gamma}$$

$$G'(\gamma) = \frac{g}{M_e} \sum_{n=0}^{\infty} (n+2h_n'-(n+1)k_n') P_n(\cos\gamma) \quad .$$

Similar expressions can be obtained for the non-zero elements of the strain tensor at the earth's surface. As pointed out by Farrell, these Green's functions are slowly convergent series and must be summed to large values of n .

The summed values for these Green's functions as a function of angle have been obtained from table values given by Farrell, where values are available for U', V', T', G' and $S'_{\gamma\gamma}$, where S' is the strain tensor. The other diagonal components of the strain tensor (off-diagonals are zero for the symmetric point load coordinate system, with load at the pole, for which the Green's functions are derived) are calculated from

$$S'_{\lambda\lambda} = \frac{U'}{a} + \cot\theta \frac{V'}{a}$$

$$S'_{rr} = -\frac{\lambda(a)}{\sigma(a)} (S'_{\gamma\gamma}+S'_{\lambda\lambda})$$

where $\lambda(a)$ and $\sigma(a)$ are the Lamé parameters at the top layer of the earth model. Here primes on the Green's functions denote that they are in the symmetric point load coordinate system.

The earth responses to ocean tides are then

$$R(\phi,\lambda;t) = \iint R^2 \, \xi(\phi',\lambda';t) \, \rho G_f(\gamma) d\Omega'$$

where

$$\cos\gamma = \sin\phi\sin\phi' + \cos\phi\cos\phi'\cos(\lambda-\lambda')$$

and $\xi(\phi,\lambda;t)$ denotes the ocean tide. Here $G_f(\gamma)$ represents the Green's function discussed above transformed to an earth-fixed coordinate system and R represents the appropriate response. The global ocean tide models developed for GSFC and available in the CALTOR program are those used in the present work. The total tide is approximated as

$$\xi(\phi,\lambda;t) = \sum_i \xi_i\ (\phi,\lambda;t)$$

where

$$\xi_i(\phi,\lambda;t) = A_i(\phi,\lambda)\ \cos[\sigma_i(t-t_o)+\psi_i(\phi,\lambda)+\varepsilon_i]$$

and $i = M_2, S_2, N_2, K_2, K_1, O_1, P_1$. The total response is then

$$R(\phi,\lambda;t) = \sum_i R_i(\phi,\lambda;t)$$

where

$$R_i(\phi,\lambda;t) = P_{i_{G_f}}\ \cos\sigma_i(t-t_o) - Q_{i_{G_f}}\ \sin\sigma_i(t-t_o)$$

and

$$P_{i_{G_f}}\ (\phi,\lambda) \equiv \iint R^2\rho A_i(\phi',\lambda')\cos[\psi_i(\phi',\lambda')+\varepsilon_i]G_f(\gamma)\sin\phi' d\phi' d\lambda'$$

$$Q_{i_{G_f}}\ (\phi,\lambda) \equiv \iint R^2\rho A_i(\phi',\lambda')\sin[\psi_i(\phi',\lambda')+\varepsilon_i]G_f(\gamma)\sin\phi' d\phi' d\lambda' \quad .$$

To evaluate the convolutions, it must be realized that the primed Green's functions described earlier are with respect to the symmetric point load coordinate system, and to resolve components of vector and tensor quantities appropriate transformations must be applied. Let (ϕ,λ) be the latitude and longitude of the point of evaluation for the convolution and (ϕ',λ') be the latitude and longitude of the water column being considered as the load. Moreover, let $\hat{r}$, $\hat{\theta}$, $\hat{\lambda}$ denote the unit vectors in the direction of increasing r, θ, λ at the point of evaluation (where θ is the co-latitude). Then the Green's function for the horizontal displacement in the $\hat{\theta}$ and $\hat{\lambda}$ directions, respectively, are

$$V_\theta(\gamma) = V'(\gamma)\cos\alpha$$

$$V_\lambda(\gamma) = V'(\gamma)\sin\alpha$$

where

$$\cos\alpha = \frac{\sin\phi' - \sin\phi\cos\gamma}{\cos\phi\sin\gamma}$$

$$\sin\alpha = \frac{\cos\phi'\sin(\lambda-\lambda')}{\sin\gamma}$$

and as before

$$\cos\gamma = \sin\phi\sin\phi' + \cos\phi\cos\phi'\cos(\lambda-\lambda')$$

$$\sin\gamma = \sqrt{1-\cos^2\gamma} \quad .$$

Similar expressions for the radial displacement and strain tensor components are

$$U(\gamma) = U'(\gamma)$$

$$S_{rr} = S'_{rr}$$

$$S_{\theta\theta} = \cos^2\alpha\ S'_{\gamma\gamma} + \sin^2\alpha\ S'_{\lambda\lambda}$$

$$S_{\lambda\theta} = \sin\alpha\cos\alpha\ (S'_{\gamma\gamma} - S'_{\lambda\lambda})$$

$$S_{\lambda\lambda} = \sin^2\alpha\ S'_{\gamma\gamma} + \cos^2\alpha\ S'_{\lambda\lambda} \quad .$$

Software is presently complete for evaluating the convolutions for the following responses:

radial displacement, $R[U_i(\phi,\lambda;t)]$

eastward displacement, $R[V_{\lambda_i}(\phi,\lambda;t)]$

southward displacement, $R[V_{\theta_i}(\phi,\lambda;t)]$

rr Strain Tensor component, $R[S_{rr_i}(\phi,\lambda;t)]$

$\theta\theta$ Strain Tensor component, $R[S_{\theta\theta_i}(\phi,\lambda;t)]$

$\lambda\theta$ Strain Tensor component, $R[S_{\lambda\theta_i}(\phi,\lambda;t)]$

$\lambda\lambda$ Strain Tensor component, $R[S_{\lambda\lambda_i}(\phi,\lambda;t)]$

where $i = M_2, S_2, N_2, K_2, K_1, O_1, P_1$. The output of the software is an amplitude, $A_i(\phi,\lambda)$, and phase, $\Psi_i(\phi,\lambda)$, defined at each 3°×3° grid point over the surface of the globe. The response is then

$$R_i(\phi,\lambda;t) = A_i(\phi,\lambda)\ cos[\sigma_i(t-t_o)+\Psi_i(\phi,\lambda)+\varepsilon_i]$$

The amplitude and phase have been computed for the global radial displacement due to M_2 and are displayed in Figure 2.

2.3 Conformance to Schedule

Work is currently behind projected schedule. This is due primarily to the slow and unpredictable turn-around on the GSFC 360-91 using STAND-BY computer time. Program development work is done on the 360-95 when practicable, but the 360-91 is more efficient for the high-core requirements of the programs for execution. At this point in the work, we had anticipated evaluation of responses for the radial and tangential displacements, tilt, gravity acceleration and strain tensor components due to the M_2 tide. Currently we have only the radial displacements for the M_2. We anticipate better turn-around during the holiday season and some of the delay in schedule can be made up.

2.4 Analysis of Progress

With the exception of delayed turn-around detailed in Section 2.3, progress was satisfactory during the reporting period. The computer programs were developed and checked-out, and the body tide radial displacement subroutines are suitable for stand-alone use. The amplitude and phase chart for the radial response to the M_2 tide loading is also of interest as a partial result.

2.5 Efforts to Achieve Reliability

Not applicable.

2.6 Personnel Changes

There have been no personnel changes during the reporting period. Persons involved in the work to date are

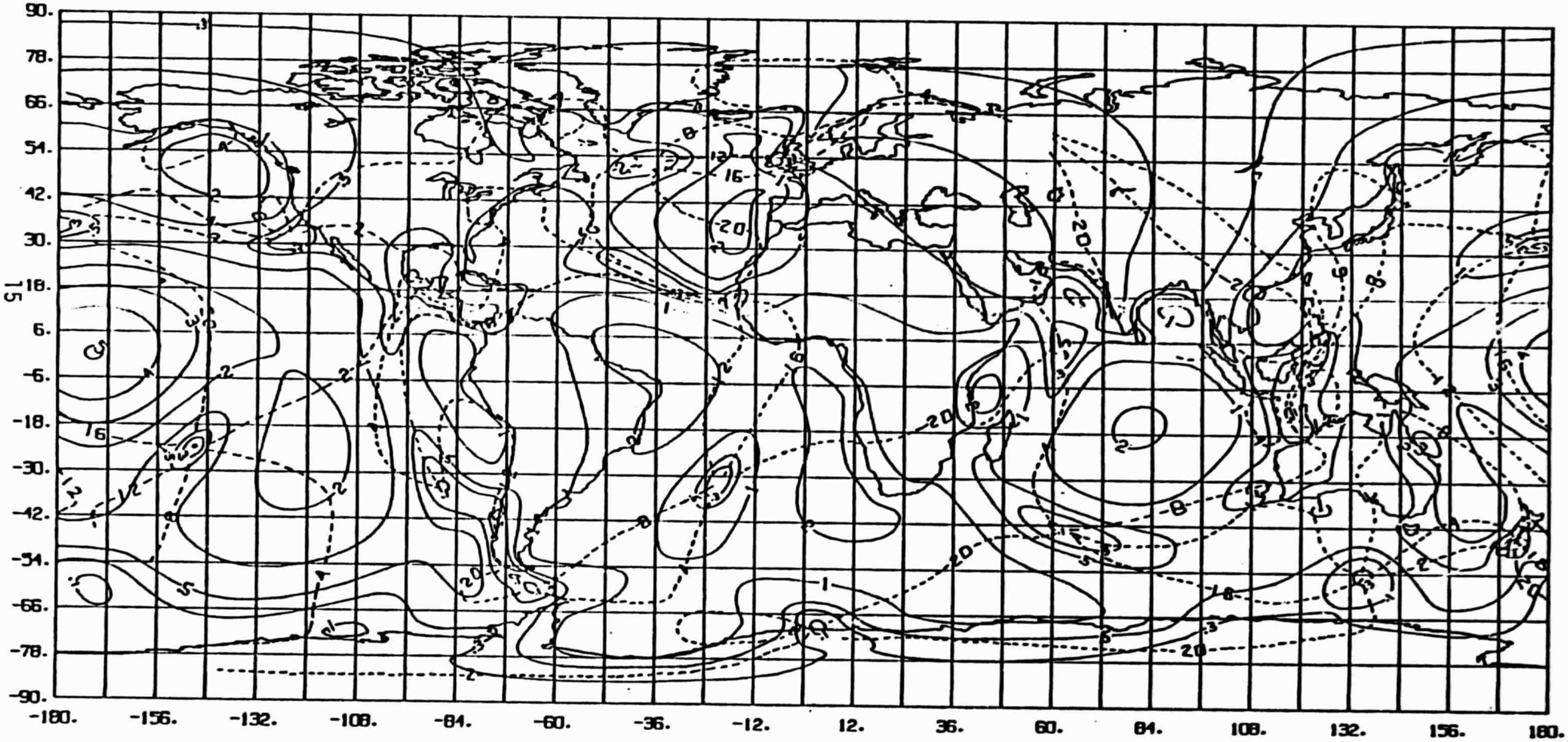
Figure 2

CRUSTAL UPLIFT DUE TO M_2 OCEAN TIDE LOADING

$$u(\phi,\lambda;t) = A(\phi,\lambda)\cos[\sigma_1 t+\psi(\phi,\lambda)]$$

$A(\phi,\lambda)$ ——— SOLID LINES (CM.)

$\psi(\phi,\lambda)$ ------ DASHED LINES (HOURS)



Ronald Estes - Principal Investigator
Daniel Chin - Senior Programmer/Analyst
Jim Strayer - Programmer

2.7 New Technology

There has been no new technology developed, as we understand the meaning of the new technology clause, during the reporting period.

3.0 WORK PLANNED FOR NEXT REPORTING PERIOD

(A) The convolution evaluation program will be run for the M_2, S_2, K_1 constituents to produce amplitudes and phases on a global 3°×3° grid for:

(1) radial displacements (M_2 already complete)
(2) horizontal displacements (east and south components)
(3) strain tensor components
(4) tilt and gravity acceleration.

(B) The amplitudes and phases for the earth responses will be expanded into a series of spherical harmonics in the form

$$A\cos\psi = \sum_{n,m} [a_{nm}\cos m\lambda + b_{nm}\sin m\lambda]\, P_{nm}(\cos\phi)$$

$$A\sin\psi = \sum_{n,m} [c_{nm}\cos m\lambda + d_{nm}\sin m\lambda]\, P_{nm}(\cos\phi) \quad .$$

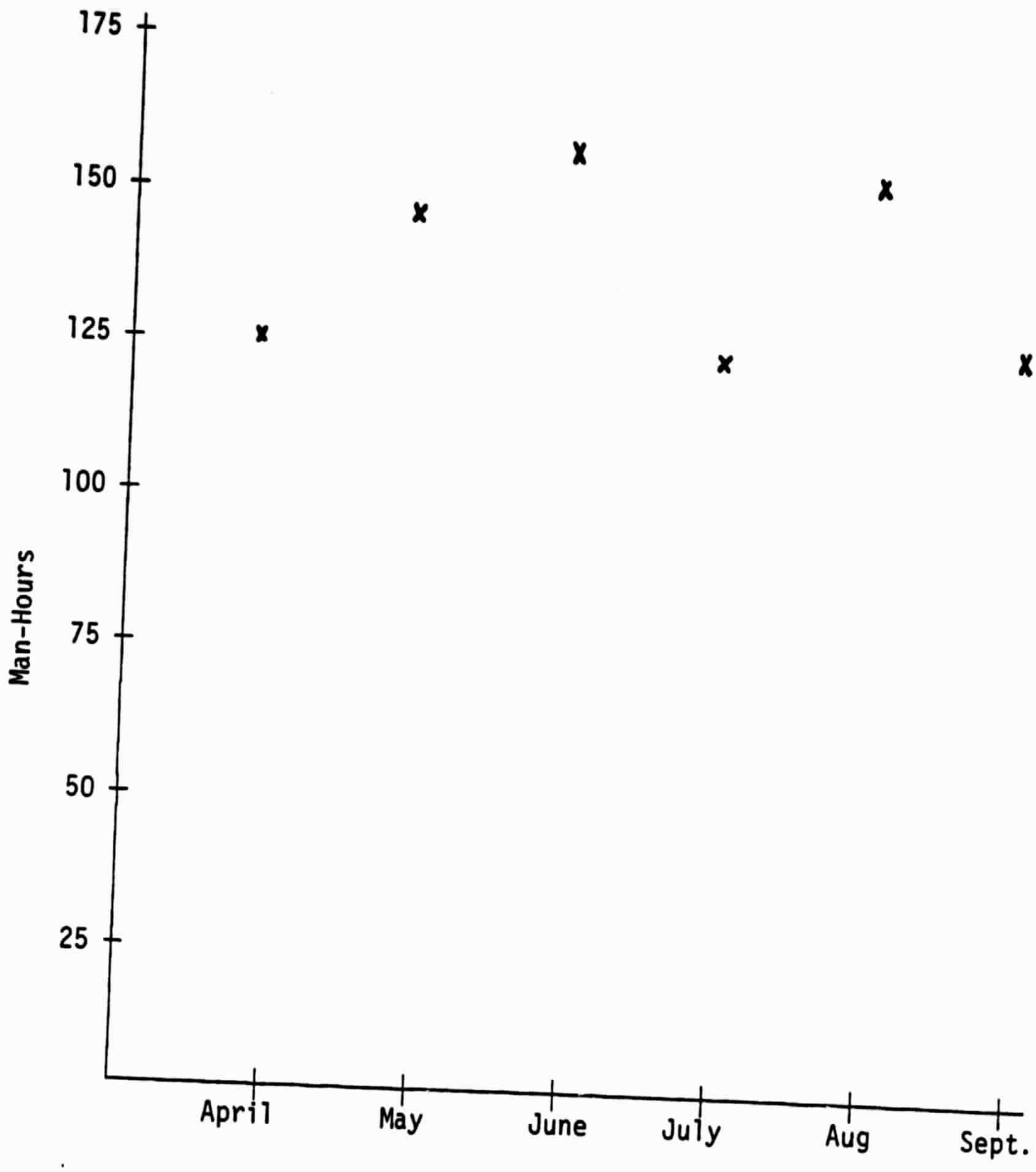
A special purpose program will be fabricated to perform this calculation.

(C) A software system will be fabricated to evaluate the radial displacement due to ocean loading and the body tide as a function of ϕ,λ and t . Effects due to M_2, S_2, and K_1 will be included. The calculations will be in terms of spherical harmonic coefficients.

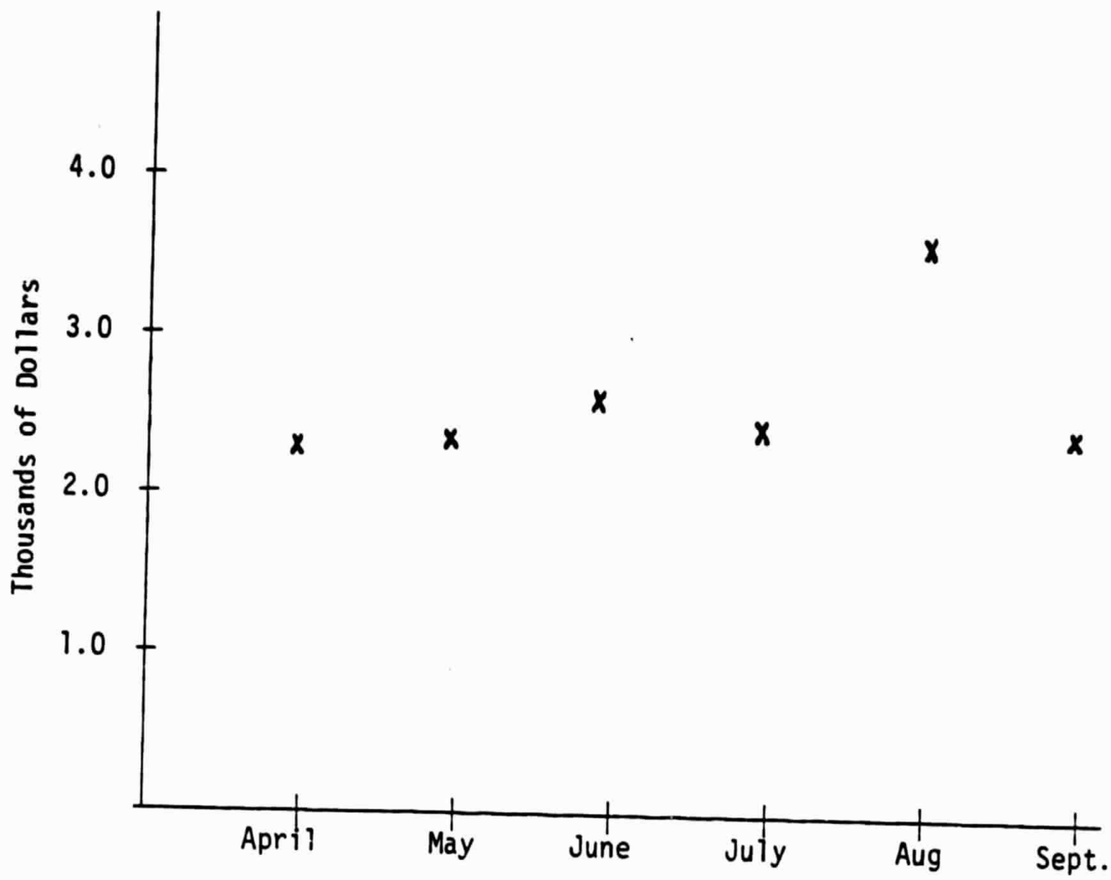
(D) The software system described in (C) will be designed such that the user may substitute the spherical harmonic expansion coefficients for any of the earth responses in place of those for the radial displacement and obtain the response as a function of ϕ,λ and t .

4.0 FISCAL AND PLANNING DATA

(A) Man-hours Expenditure Chart



(B) Funds Expenditure Chart



C. Funds Expenditure Report (Semi-Annual)

Total Task	26.8	
Prior Expenditure		0.0
Expenditure this Period		15.2
Total Expenditure to Date		15.2
Amount Remaining	11.6	
Total Hours Expended	824	
Amount Next Period	11.6	
Amount Require to Complete Work	11.6	